Comment on "Next-to-next-to-leading order vacuum polarization function of heavy quark near threshold and sum rules for $b\bar{b}$ system" and "Next-to-next-to-leading order relation between $R(e^+e^- \to b\bar{b})$ and $\Gamma_{\rm sl}(b \to cl\nu_l)$ and precise determination of $|V_{cb}|$"

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Abstract

The most recent recalculation of the two-loop correction to the static quark-antiquark potential gave the numerical value different from the previously known one. We comment on the effect this change produces on the numerical estimates of the bottom quark pole mass m_b, the strong coupling constant α_s and the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ obtained in our papers [1, 2].

In two recent papers [1, 2] numerical values of the bottom quark pole mass m_b, the strong coupling constant α_s and the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ have been determined from the sum rules for the Υ system and the B-meson semileptonic width. These phenomenological results have been obtained by exploiting the next-to-next-to-leading order expression for the vacuum polarization function of a heavy quark near the threshold. This expression depends on the value of the two-loop correction to the static quark-antiquark potential. In the analyses of refs. [1, 2] the numerical value of coefficient a_2 obtained in [3] was used. Recently the two-loop correction to the static potential has been recalculated in [4] with a new result for the coefficient a_2 that differs from the previous one. However we found that the use of corrected numerical value of the coefficient a_2 leads to a change of the numerical estimates for m_b, α_s and $|V_{cb}|$ obtained in our papers that lies well within the error bars given for these parameters in [1, 2].

In ref. [1] we applied the sum rules technique for the system of Υ resonances to determine the values of the bottom quark pole mass m_b and strong coupling constant α_s. The analysis is based on the result for the heavy quark polarization function near the threshold in the next-to-next-to-leading order of perturbative QCD and relativistic expansion. This result depends, in particular, on the two-loop correction to the static potential of the quark-antiquark interaction first computed in [3]. Recently this correction has been recalculated independently with another technique in [4]. A different value of the coefficient a_2 came out

$$a_2 = \left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^4}{4} + \frac{22}{3}\zeta(3)\right) C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta(3)\right) C_A T_F n_f$$
$$- \left(\frac{55}{3} - 16\zeta(3)\right) C_F T_F n_f + \left(\frac{20}{9} T_F n_f\right)^2$$

which is smaller than the previous result of ref. [3] by an amount $2\pi^2 C_A^2$. After performing the analysis with the corrected value we found that this variation of the coefficient affects our numerical estimates only slightly. Namely, the value of α_s extracted from the sum rules is practically insensitive to the above variation while the value of m_b decreases for $\sim 0.1\%$ when the corrected two-loop coefficient is used

instead of the previous one. Since the theoretical uncertainty in m_b exceeds 1% this variation is negligible.

In the paper [2] we used the relation between the moments of the Υ system spectral density and the inclusive B-meson semileptonic width for precise determination of the $|V_{cb}|$ matrix element. Evaluating the moments we used the next-to-next-to-leading order expression [1] of the heavy quark polarization function near the threshold computed with Peter's coefficient a_2. Changing it to the correct Schröder's value we obtain $\sim 0.1\%$ increase of the extracted numerical value for $|V_{cb}|$. The total theoretical uncertainty of this quantity exceeds 3% that makes this variation completely negligible.

The reason for such a weak influence of the change on our results is that the coefficient a_2 parameterizes only a part of the correction to the NRQCD Hamiltonian in this order that makes the dependence on a_2 much softer than one could expect from the direct numerical change of the coefficient itself.

To conclude, the numerical estimates of the bottom quark pole mass, the strong coupling constant and the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ presented in refs [1, 2] are insensitive to the correction [4] of the previously obtained value of the two-loop coefficient a_2 [3]. The corresponding shifts of the extracted values of m_b, α_s and $|V_{cb}|$ are an order of magnitude smaller than the theoretical uncertainties of these quantities given in [1, 2].

References

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